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Discussion

Comments on "Explicit transient solutions for a mode III crack subjected to dynamic concentrated loading in a piezoelectric material" by Yi-Shyong Ing, Mau-Jung Wang [Int. J. Solids Struct. 41 (2004) 3849–3864]

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Abstract

In this comment it is pointed out that the analysis of the dynamic stress intensity factor, dynamic electric displacement intensity factor and dynamic energy release rate conducted by Ing and Wang [Ing, Y.S., Wang, M.J., 2004. Explicit transient solutions for a mode III crack subjected to dynamic concentrated loading in a piezoelectric material. International Journal of Solids and Structures 41, 3849–3864] is incorrect. The correct analysis and corresponding correct plots are presented.

Keywords: Crack; Piezoelectric material; Dynamic response; Fracture; Wave propagation

Even simple calculations show that results presented by Ing and Wang (2004) are incorrect. It is easy to see that from Eqs. (64), (66) and Table 1 it follows that for time $t > b_{bg}h$

$$K_{\mathrm{III}}^{(D)}(t)\frac{h^{1/2}e_{15}}{p\varepsilon_{11}} = K_{\mathrm{III}}^{(D),s}\frac{h^{1/2}e_{15}}{p\varepsilon_{11}} = \frac{k_e^2}{1-k_e^2}\sqrt{\frac{2}{\pi}} \approx \begin{cases} 0.778, & \text{for PZT-4 material} \\ 0.705, & \text{for PZT-5 material} \\ 0.239, & \text{for BaTiO}_3\text{ material} \end{cases}$$

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so that $K_{\mathrm{III}}^{(D)}(t)\frac{h^{1/2}e_{15}}{p\varepsilon_{11}} > 0.7$ for PZT-4 and PZT-5 materials, when $t > b_{bg}h$, however on Fig. 6 it is shown that $K_{\mathrm{III}}^{(D)}(t)\frac{h^{1/2}e_{15}}{p\varepsilon_{11}} < 0.5$ for these materials in any time interval.

In the paper by Ing and Wang (2004)

(a) it is concluded that the dynamic stress intensity factor, dynamic electric displacement intensity factor and the dynamic energy release rate are equal to zero, when $t < bh$;
(b) it is concluded that direct calculations cannot be applied to the integrals representing dynamic intensity factors for $t < b_{bg}h$ (in the discussion following Eqs. (61) and (62) it is told that direct calculations cannot be applied to the integrals (61) and (62) for $t > b_{bg}h$, however the next sentence and the further text make it obvious that it is a typesetting mistake and that that conclusion relates to $t < b_{bg}h$, but not to $t > b_{bg}h$);
(c) on Figs. 5–7 plots are presented describing the behaviours of the dynamic stress intensity factor, the dynamic electric displacement intensity factor and the dynamic energy release rate.

In this comment it will be shown that the conclusions (a) and (b) and the plots on Figs. 5–7 are incorrect. The correct analysis and plots will be presented.

Deforming the path of integration in (59) from Γ_λ to the one along the branch cut $\{\operatorname{Im}\lambda = 0, \operatorname{Re}\lambda < 0\}$ we obtain that

$$\bar{K}_{\mathrm{III}}^{(\tau)}(s) = \frac{p}{\pi}\sqrt{\frac{2}{h}}\frac{1}{\sqrt{s}}\int_0^\infty \operatorname{Re}\left[\frac{\sqrt{\tau + \mathrm{i}0 - bh}}{Q_-((\tau + \mathrm{i}0)/h)}\right]\frac{\mathrm{e}^{-s\tau}}{\tau - b_{bg}h}\,\mathrm{d}\tau \tag{1}$$

which, after applying the inverse Laplace transform becomes

$$\begin{aligned} K_{\mathrm{III}}^{(\tau)}(t) &= p\sqrt{\frac{2}{\pi^3 h}}\int_0^t \operatorname{Re}\left[\frac{\sqrt{\tau + \mathrm{i}0 - bh}}{(\tau - b_{bg}h)Q_-((\tau + \mathrm{i}0)/h)\sqrt{t - \tau}}\right]\mathrm{d}\tau H(t) \\ &= -p\sqrt{\frac{2}{\pi^3 h}}\int_0^t \operatorname{Im}\left[\frac{\sqrt{\tau + \mathrm{i}0 - bh}}{(\tau - b_{bg}h)Q_-((\tau + \mathrm{i}0)/h)\sqrt{\tau + \mathrm{i}0 - t}}\right]\mathrm{d}\tau H(t). \end{aligned} \tag{2}$$

As the functions $\sqrt{\tau - bh}$, $\sqrt{\tau - t}$ and $Q_-(\tau h^{-1})$ are analytic in the entire complex τ-plane cut respectively along $\{\operatorname{Im}\tau = 0, \operatorname{Re}\tau \leqslant bh\}$, $\{\operatorname{Im}\tau = 0, \operatorname{Re}\tau \leqslant t\}$ and $\{\operatorname{Im}\ \tau = 0, \operatorname{Re}\tau \in [0, bh]\}$, the additive term $+\,\mathrm{i}0$ is introduced in (1) and (2) to show, that τ is located on the upper side of the branch cuts.

The integrand of (2) has a first-order singularity at $\tau = b_{bg}h$ and branch points at $\tau = 0$, $\tau = bh$ and $\tau = t$.

For $t < bh$ a direct evaluation procedure for the stress intensity factor $K_{\mathrm{III}}^{(\tau)}(t)$ cannot be applied to the integral, however, using the identity

$$Q_-(\lambda)Q_+(\lambda) = (1 + k_e^2)\frac{\alpha(\lambda)}{\alpha(\lambda) + k_e^2\beta(\lambda)}$$

the expression (2) can be written in the following form:

$$K_{\mathrm{III}}^{(\tau)}(t) = \frac{k_e^2 p}{1 + k_e^2}\sqrt{\frac{2}{\pi^3 h}}\int_0^t \frac{\tau Q_+(\tau/h)}{(\tau - b_{bg}h)\sqrt{t - \tau}\sqrt{bh + \tau}}\,\mathrm{d}\tau H(t) \tag{3}$$

which does not contain any branch cut along the path of integration and is more convenient for numerical calculations.

For $t > bh$, the integrand of (2) is analytic in the entire τ- plane cut along $\{\operatorname{Im}\tau = 0, \operatorname{Re}\tau \in [0, t]\}$, except for the pole point at $\tau = b_{bg}h$. In this case the path of integration can be closed around the branch cut as shown in Fig. 1. As a result the integral (2) takes the form

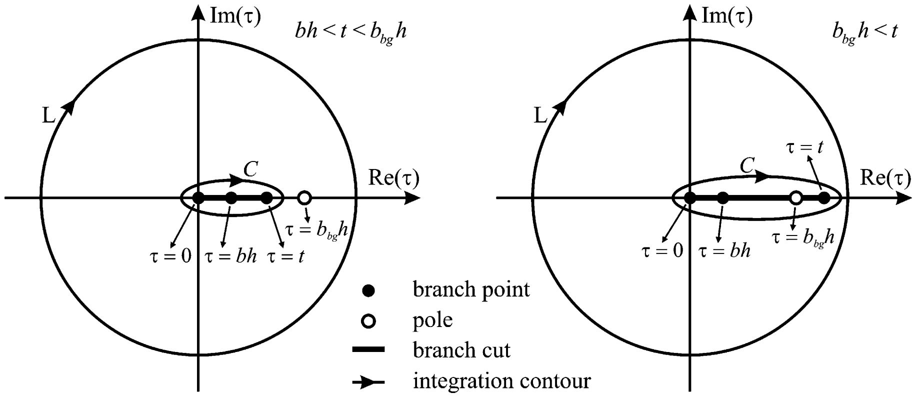

Fig. 1. The integration path C for different time intervals.

$$K_{\rm III}^{(\tau)}(t) = -p\sqrt{\frac{2}{\pi^3 h}}\frac{1}{2{\rm i}}\oint_C \frac{\sqrt{\tau - bh}}{(\tau - b_{bg}h)Q_-(\tau/h)\sqrt{\tau - t}}{\rm d}\tau H(t) \tag{4}$$

where C is a counter-clockwise contour, which embraces the branch cut $\{{\rm Im}\,\tau = 0, {\rm Re}\,\tau \in [0,t]\}$ of the integrand.

Applying the Cauchy's integral formula we now conclude that

if $bh < t < b_{bg}h$, then

$$K_{\rm III}^{(\tau)}(t) = p\sqrt{\frac{2}{\pi h}}\left[1 - \frac{\sqrt{b_{bg} - b}}{Q_-(b_{bg})\sqrt{b_{bg} - h^{-1}t}}\right] \tag{5}$$

if $t > b_{bg}h$, then

$$K_{\rm III}^{(\tau)}(t) = p\sqrt{\frac{2}{\pi h}}. \tag{6}$$

Summing up Eqs. (3), (5) and (6) we obtain the following final expression for the stress intensity factor:

$$K_{\rm III}^{(\tau)}(t) = p\sqrt{\frac{2}{\pi h}}\left\{\frac{k_e^2}{1+k_e^2}\frac{1}{\pi}\int_0^t \frac{\tau}{\sqrt{t-\tau}\sqrt{bh+\tau}}\frac{Q_+(\tau/h)}{\tau - b_{bg}h}{\rm d}\tau[H(t) - H(t-bh)] + H(t-bh)\right.$$
$$\left. - \frac{\sqrt{b_{bg} - b}}{Q_-(b_{bg})\sqrt{b_{bg} - h^{-1}t}}[H(t-bh) - H(t - b_{bg}h)]\right\}. \tag{7}$$

Then, according to Eqs. (59), (60) and (73) of the paper Ing and Wang (2004) we have that

$$K_{\rm III}^{(D)}(t) = \frac{\varepsilon_{11}k_e^2}{e_{15}(1-k_e^2)}K_{\rm III}^{(\tau)}(t) \tag{8}$$

$$G(t) = \frac{1}{2\bar{c}_{44}(1-k_e^2)}[K_{\rm III}^{(\tau)}(t)]^2 \tag{9}$$

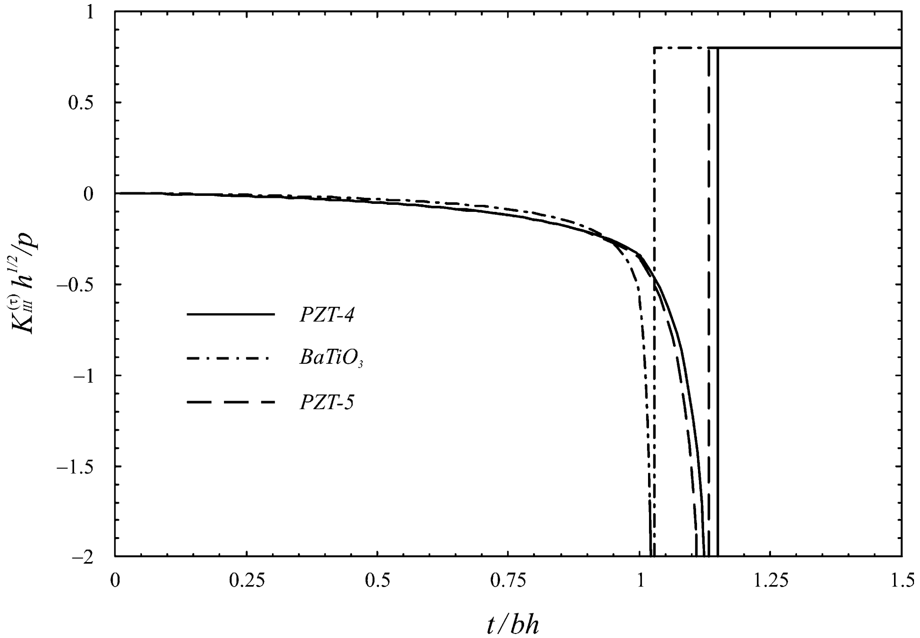

Fig. 2. Normalized dynamic stress intensity factors versus normalized time for various piezoelectric materials.

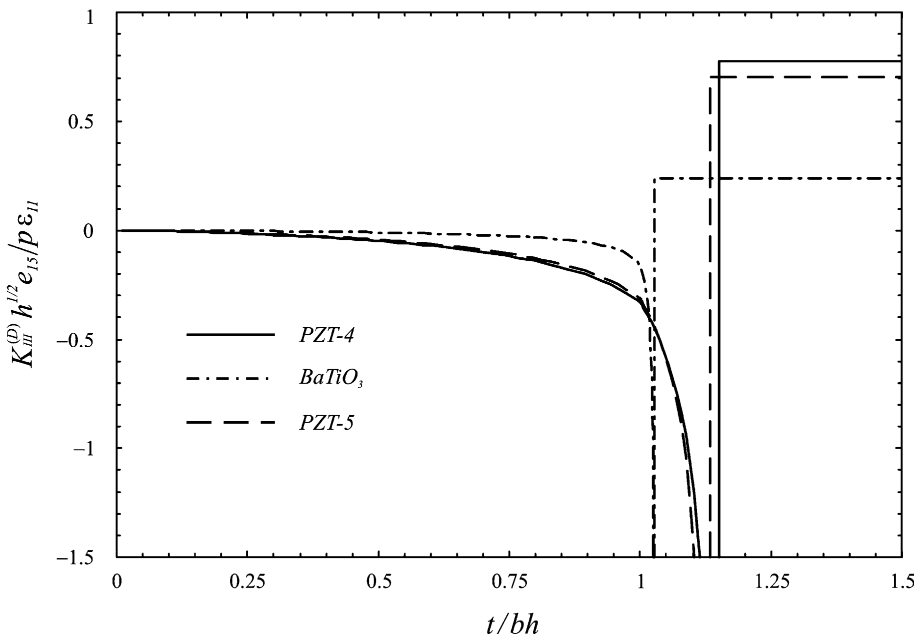

Fig. 3. Normalized dynamic electric displacement intensity factors versus normalized time for various piezoelectric materials.

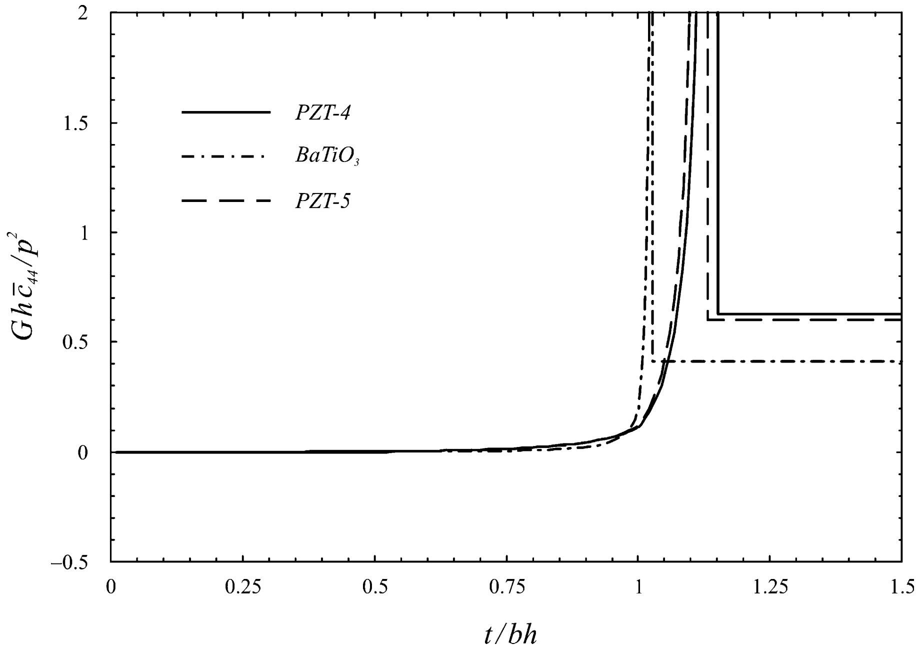

Fig. 4. Normalized dynamic energy release rates versus normalized time for various piezoelectric materials.

Using (7)–(9) numerical calculations can be evaluated. The variation of the dynamic stress intensity factors, dynamic electric displacement intensity factors and dynamic energy release rates with the normalized time $t/(bh)$ are shown in Figs. 2–4. The calculations are conducted for the piezoelectric materials PZT-4, $BaTiO_3$ and PZT-5, which have been chosen by Ing and Wang (2004) in their paper.

The expressions (7)–(9) and Figs. 2–4 show that

(a) dynamic intensity factors and the dynamic energy release rates are *not* equal to zero, when $t < bh$;
(b) direct calculations *can* be applied to the integrals representing dynamic intensity factors for $t > bh$, and particularly for $t \in [bh, b_{bg}h]$;
(c) the dynamic stress intensity factor, the dynamic electric displacement intensity factor and the dynamic energy release rate have the behaviours shown in Figs. 2–4, which are correct and different from Figs. 5–7 introduced by Ing and Wang (2004).

Reference

Ing, Y.S., Wang, M.J., 2004. Explicit transient solutions for a mode III crack subjected to dynamic concentrated loading in a piezoelectric material. International Journal of Solids and Structures 41, 3849–3864.